

## *CP* Violation in *B* mesons

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### Abstract

Symmetries and their conservation laws play a fundamental role in Physics. Among them, the discrete symmetries corresponding to charge (*C*), parity (*P*), and time (*T*) transformations are extensively used in the theory of the elementary particles and their interactions (so called Standard Model (SM)) to give the basis of the fundamental physical description of nature. Eventual discoveries of violations of these symmetries become a crucial test for our understanding of the nature. It was assumed that the three discrete symmetries were not violated until 1956 when it was found that *P* is violated in the weak interaction. Soon it was understood that also the *C* is violated in the weak interaction. At that time these two violated symmetries were replaced by their combination, *CP*, which was considered a new fundamental symmetry. In 1964 also the *CP* was found violated in the case of the neutral *K* meson system. Since that year there were many achievements in theories and experiments in order to explain this symmetry violation. In the last five years the main contribution comes from the discovery of the *CP* violation in *B* meson system. In this note we will describe briefly how the *CP* violation is described in the SM and the main experimental results obtained in the *B* mesons system.

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# 1 Introduction

Symmetries in physical systems refer to the properties that the systems exhibit under certain transformations. In particular we look to the aspects which are unchanged after the transformation, according to a particular observation. In this sense a symmetry is a physical or mathematical feature of the system (observed or intrinsic) that is “preserved” under some change. A real-world example is the rotation of a spherical ball: when rotated about its centre, it will appear exactly as it did before the rotation. The ball is said to exhibit *spherical symmetry*. The transformations may be continuous (such as rotations), characterized by a continuous change in the geometry of the system, or discrete (such as reflections), described by non-continuous changes, and lead to corresponding types of symmetries. These symmetries are usually formulated mathematically and can be exploited to simplify many problems.

The set of operators on the Hilbert space of state functions on the quantum field contains both discrete and continuous transformations that preserve the Minkowski interval  $t^2 - \vec{x}^2$ . The set of continuous transformations that preserve this interval are the familiar Lorentz transformations, comprised of the product space of rotations, translations, and Lorentz boosts. The three independent discrete transformations that also preserve  $t^2 - \vec{x}^2$  are the charge conjugation operator ( $C$ ), the parity operator ( $P$ ), and the time-reversal operator ( $T$ ). These form a complete set of discrete Minkowski interval-preserving transformations of the Hilbert space.

## 1.1 Parity

The parity operator  $P$  reverses the signs of the 3 spatial elements of a four-vector:  $(t, \vec{x}) \rightarrow (t, -\vec{x})$  and  $(E, \vec{p}) \rightarrow (E, -\vec{p})$ . One can easily visualize parity as a mirror-image plus an 180-degree rotation normal to the plane of the mirror — this reverses the momentum of a particle but leaves its spin unchanged. Consider the action of parity on the particle and antiparticle annihilation operators of the Dirac field  $a_p^s$  and  $b_p^s$ . Parity transforms the states  $a_p^s|0\rangle$  and  $b_p^s|0\rangle$  to  $a_{-p}^s|0\rangle$  and  $b_{-p}^s|0\rangle$ . This implies  $Pa_p^sP^{-1} = \eta_a a_{-p}^s$  and  $Pb_p^sP^{-1} = \eta_b b_{-p}^s$ , where  $\eta_a$  and  $\eta_b$  are phases. Since  $P^2 = \mathbf{1} \Rightarrow \eta_a, \eta_b$  must equal  $\pm 1$  (the parity group, as with the other two discrete operators, is idempotent, *i. e.*  $P^{-1} = P$ ).

In 1956, Lee and Yang showed that parity conservation, while well-tested in strong and electromagnetic interactions, was not experimentally constrained for weak interactions, and proposed a list of experimental tests.[1] C. S. Wu and collaborators performed one of these experiments, and showed that parity was not conserved in nuclear  $\beta$  decay, marking the inception of the discovery of the symmetry-violating properties in weak interaction.[2]

## 1.2 Charge Conjugation

The charge conjugation operator  $C$  is defined to be the transformation of a particle into its antiparticle without changing momentum or spin. Thus  $Ca_p^sC^{-1} = \eta'_a b_p^s$  and  $Cb_p^sC^{-1} = \eta'_b a_p^s$ .

## 1.3 Time Reversal

The time reversal operator  $T$  reverses momentum and spin and also flips the sign of the time component of a state. Therefore we want the transformation of the Dirac particle and antiparticle annihilation operators to be  $Ta_p^sT^{-1} = \eta'_a a_{-p}^{-s}$  and  $Tb_p^sT^{-1} = \eta'_b b_{-p}^{-s}$ .

## 1.4 CP Symmetry

If  $P$  was found violated in 1956, the  $CP$  transformation, obtained combining both  $P$  and  $C$  operators, was still considered valid. The discovery, eight years later, of the decay of the neutral kaon meson with long lifetime in two pions by Christenson, Cronin, Fitch and Turlay establishes the  $CP$  violation in the weak interaction.[3]

The neutral strange kaon mesons ( $K^0$  with mass of about  $498 \text{ MeV}/c^2$  [4]) have the peculiarity, as pointed for the first time by Gell-Mann and Pais in 1955[5], that the quantum number distinguishing particle  $K^0$  and antiparticle  $\bar{K}^0$  (strangeness, being  $+1$  for the former and  $-1$  for the latter) is not conserved by weak interactions, so that the two would be coupled through them, and  $K^0 \leftrightarrow \bar{K}^0$  transitions would be allowed through the common (virtual) final states. If we consider  $CP$  a valid symmetry, we have  $CP|K^0\rangle = |\bar{K}^0\rangle$  and  $CP|\bar{K}^0\rangle = |K^0\rangle$  and the symmetry can be used to characterize the physical states, which are therefore

$$|K_1\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle + |\bar{K}^0\rangle), \quad (1)$$

$$|K_2\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle - |\bar{K}^0\rangle), \quad (2)$$

for which

$$CP|K_1\rangle = +|K_1\rangle, \quad (3)$$

$$CP|K_2\rangle = -|K_2\rangle. \quad (4)$$

A first evidence of the two physical states was given by an experiment in the Brookhaven laboratory in 1956[6] with two neutral  $K$  mesons, now called short-lived ( $K_S^0 \equiv K_1$ ) and long-lived ( $K_L^0 \equiv K_2$ ), whose lifetimes are  $\tau(K_S^0) = 0.89 \cdot 10^{-10} \text{ s}$  and  $\tau(K_L^0) = 5.17 \cdot 10^{-8} \text{ s}$ . The large lifetime difference among the two states originates largely from phase space volume: the only hadrons lighter than the  $K$  are pions, and the ratio of the kaon mass to the pion mass is small enough that decays into two or three pions have a large difference in  $Q$ -values.

It is easy to understand that a  $2\pi$  final state of  $K$  decays has  $CP = +1$ , while a  $3\pi$  final state has  $CP = -1$ . If  $CP$  is conserved, then we can have only  $K_S^0 \rightarrow 2\pi$  and  $K_L^0 \rightarrow 3\pi$ . In 1963, a bubble chamber experiment by R. Adair and collaborators showed an anomalous excess of  $2\pi$  events from decays of a  $K_L^0$  beam.[7] The effect was statistically significant, but not enough for a discovery. A proposal for a new experiment to clarify the situation was submitted at Brookhaven by J. Cronin, V. Fitch and R. Turlay in April 1963. In one month the experiment was approved and in June 2nd, 1963, the three physicists, joined by J. Christenson, started the 40-day data-taking period. After 6 months of checks of the results, in order to exclude any possible spurious effects, the discovery of the  $CP$  symmetry violation was published[3], reporting the decay ratio:

$$\frac{\Gamma(K_L^0 \rightarrow \pi^+\pi^-)}{\Gamma(K_L^0 \rightarrow \text{charged})} = (2.0 \pm 0.4) \cdot 10^{-3}. \quad (5)$$

## 1.5 The Cosmological Issue

As known from different types of astronomical data, the universe is strongly charge asymmetric: it is only populated with particles, while antiparticles are practically absent. A small number of the observed antiprotons or positrons in cosmic rays can be explained by their secondary origin

through particle collisions. There is no evidence of large antimatter objects (anti-stars, anti-planets or gaseous clouds of antimatter) in our galaxy. In a naive universe model we would expect to have a symmetric production of matter and anti-matter in the primordial phase. This means that a mechanism to explain the charge asymmetric universe is needed. Seemingly, the breaking of  $C$  and  $CP$  symmetries are necessary for this purpose, but this happens to be true only in the simplest versions of the theory. According to this scenario, the generally accepted mechanism of generation of a cosmological charge asymmetry is based on three famous Sakharov's conditions[8]:

1. Non-conservation of (baryonic) charge.
2. Breaking of symmetry between particles and antiparticles, both  $C$  and  $CP$ .
3. Deviation from thermal equilibrium.

## 2 The $B$ Meson System

The measurement shows 5 marks the fall of the  $CP$  symmetry. Starting from that period, many achievements, either theoretical and experimental, have been done to find evidence of  $CP$  violation in other physics systems.

In 1973 (almost 10 years later the discover of the  $CP$  violation), Kobayashi e Maskawa suggested a generalization of the quark mixing matrix, introduced by Cabibbo[9], where the  $CP$  violation in the neutral kaons can be explained using a model with three families of quarks and leptons[10] (this happened a year before even the charm quark was discovered). The quarks of the third family, called  $b$  per bottom (or beauty) and  $t$  per top, were discovered in 1977[11] and in 1994[12], respectively. The first experimental evidence of the existence of the  $b$  quarks was obtained with the observation of a  $9.5 \text{ GeV}/c^2$  dimuon resonance in 400 GeV proton-nucleus collisions[11] and evidence for a close-by second resonance[13], later followed by a third[14]. It took a higher-luminosity electron-positron collider, the Cornell Electron Storage Ring (CESR), to observe the  $\Upsilon(4S)$  fourth radial excitation at the center of mass energy of about 10.58 GeV: its larger natural width was interpreted as indication that the  $\Upsilon(4S)$  is above the threshold for *open beauty* production[15]. Soon later, in 1983, it was found evidence of hadronic decays of charged and neutral “ $b$ -flavoured”  $B$  mesons.[16]

The  $B$  mesons are pseudoscalar mesons which can be charged or neutral with the following flavor states:

- $B_d^0 = (\bar{b}d) \equiv B^0$ ,
- $B_u^+ = (\bar{b}u) \equiv B^+$ ,
- $B_s^0 = (\bar{b}s)$ ,
- $B_c^+ = (\bar{b}c)$ ,

and their charge-conjugated states. As for neutral  $K$  mesons (see section 1.4), the pair of  $B$  conjugate neutral mesons can each mix with their respective antiparticle. The ability to mix implies that the flavor eigenstates may not be equivalent to the mass eigenstates; the observed presence of mixing (into conjugate flavor-specific decays) implies that the mass and flavor eigenstates are in fact different. So, we can describe neutral  $B$  mesons in term of two physical states combination of the flavor eigenstates

$$\begin{aligned} |B_L\rangle &= p|B^0\rangle + q|\bar{B}^0\rangle, \\ |B_H\rangle &= p|B^0\rangle - q|\bar{B}^0\rangle, \end{aligned} \tag{6}$$

where the coefficients  $p$  and  $q$  satisfy the relation  $|p|^2 + |q|^2 = 1$ . The  $|B_L\rangle$  and  $|B_H\rangle$  are the lighter and heavier mass eigenstates of the time-dependent Schrödinger equation

$$i\frac{\partial}{\partial t} \begin{pmatrix} p \\ q \end{pmatrix} = \mathcal{H} \begin{pmatrix} p \\ q \end{pmatrix}, \quad (7)$$

where

$$\mathcal{H} = \mathbf{M} - i\frac{\mathbf{\Gamma}}{2}. \quad (8)$$

The  $\mathbf{M}$  and  $\mathbf{\Gamma}$  parts are  $2 \times 2$  matrices represent the mixing and decay parts, respectively, of the time dependence. The eigenvalues  $\lambda_L$  and  $\lambda_H$  of equation 8 are:

$$\lambda_L = m_L - i\frac{\Gamma_L}{2}, \quad \lambda_H = m_H - i\frac{\Gamma_H}{2}, \quad (9)$$

where  $m_L$  and  $m_H$  are the masses of the eigenstates  $|B_L\rangle$  and  $|B_H\rangle$ , respectively, and  $\Gamma_L \equiv 1/\tau_L$  and  $\Gamma_H \equiv 1/\tau_H$  their decay parts ( $\tau_{L,H}$  are the lifetimes). We define also:

$$\begin{aligned} \Delta m &= m_H - m_L, \\ \Delta \Gamma &= \Gamma_H - \Gamma_L, \\ m &= \frac{m_H + m_L}{2}, \\ \Gamma &= \frac{\Gamma_H + \Gamma_L}{2}. \end{aligned} \quad (10)$$

In this sense  $\Delta m$  represents the mixing frequency of the  $B^0 \leftrightarrow \bar{B}^0$  oscillation.

In 1983 the MAC and MARKII experiments at Stanford Linear Accelerator Center (SLAC) measured the longevity of  $B_d^0$  mesons.[17] The observed lifetime is  $\tau_{B_d} = 1.5 \cdot 10^{-12}$  s, about 1000 times longer than the naive estimate (which was beyond the reach of observation of the two experiments). This was a great news because otherwise the time dependent measurement of the  $CP$  violation would be much more difficult. In 1987 the mixing was established, with contributions from experiments at both proton-antiproton and electron-positron colliders. Some indication for  $B^0 - \bar{B}^0$  mixing, contributed by both  $B_d = (\bar{b}d)$  and  $B_s = (\bar{b}s)$ , was found by UA-1 at the  $Spp\bar{S}$  collider[18]; clear convincing evidence was first obtained by the ARGUS Collaboration at DORIS[19], at the  $\Upsilon(4S)$ , where only  $B_d$  is produced. All these measurements are consistent with the present theoretical predictions.

For neutral  $B$  mesons, in contrast with the neutral  $K$  system, the lifetime difference  $\Delta\Gamma$  between the two mass eigenstates is *small* compared with the mixing frequency due to the difference in masses  $\Delta m$ . This difference in behavior of the  $K$  and  $B$  is due to the larger mass of the  $B$  meson. This means that the greater phase space for flavor-specific decays in the  $B$  system, which dominates the partial width (in contrast to the  $K$  system), gives equivalent contributions (by  $CPT$  symmetry) to the width of both neutral  $B$  eigenstates. The resulting lack of decay suppression of either eigenstates implies nearly equivalent lifetimes. All these features make the  $B$  meson system a very promising place where to look for  $CP$  asymmetry violations.

### 3 Three Types of $CP$ Violation

Three types of  $CP$  violation can potentially be observed at  $B$  physics experiments:

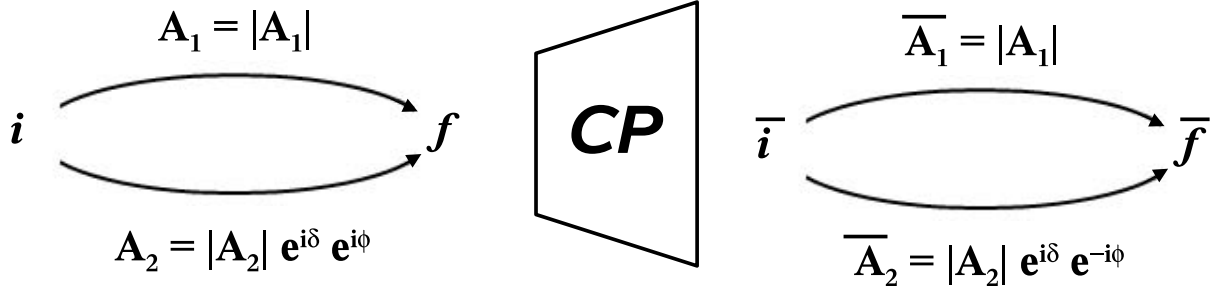


Figure 1: Effect of the “ $CP$  mirror” on interfering decay amplitudes for the transition between an initial state  $i$  and a final state  $f$ . The direct  $CP$  asymmetry is due to the interference between two amplitudes  $A_1$  and  $A_2$  with a relative  $CP$ -conserving phase  $\delta$  and a  $CP$ -violating phase  $\phi$ .

- $CP$  violation in decay (often referred to as direct  $CP$  violation);
- $CP$  violation purely in mixing;
- $CP$  violation in the interference between decays of mixed and unmixed mesons.

### 3.1 $CP$ Violation in Decay (Direct $CP$ Violation)

Direct  $CP$  violation manifests itself as a difference in the magnitude of the amplitude to a given decay as compared with its  $CP$  conjugate, thus resulting in differing rates to the two elements of the  $CP$  conjugate pair (see fig. 1). It can occur for both neutral and charged decays.<sup>1</sup> Amplitudes from  $B^0$  and  $\bar{B}^0$  to a final state  $f$  and its  $CP$  conjugate may be written as

$$A_f = \sum_i A_i e^{i(\phi_i + \delta_i)} \quad (11)$$

$$\bar{A}_{\bar{f}} = \eta_{CP} \sum_i A_i e^{i(-\phi_i + \delta_i)} \quad (12)$$

where  $\eta_{CP}$  is the  $CP$  eigenvalue (multiplied by a convention-dependent phase) if  $f$  is a  $CP$  eigenstate,  $\phi_i$  are the weak phases, and  $\delta_i$  are the strong phases.  $CP$  violation can only occur when the different weak phase contributions also have different strong phases (otherwise a simple rotation can remove the strong phase and thus the ratio would clearly have unit magnitude). It can also only occur when weak phases are nontrivial, *i. e.* when exists a relative phase between them. Only when both different weak phases *and* different strong phases are present, we may have the condition:

$$|\bar{A}_{\bar{f}}/A_f| \neq 1 \quad (13)$$

This is  $CP$  violation in decay.

More than 30 years of experimental researches in the kaon system has yielded only in 1999 the observation of direct  $CP$  violation.[20] More recently in 2004 the direct  $CP$  violation has been observed in the  $B$  system too (see section 6.3).[21]

<sup>1</sup>For charged decays, it is the *only* potential manifestation of  $CP$  violation.

### 3.2 $CP$ Violation Purely in Mixing

From the equations 6, we have that if  $q$  and  $p$  have different magnitudes, the  $CP$  conjugates of the mass eigenstates clearly will differ from the mass eigenstates themselves by more than a trivial phase. Thus the mass eigenstates will not be  $CP$  eigenstates and  $CP$  violation will be manifest.  $CP$  violation from

$$|q/p| \neq 1 \quad (14)$$

is purely an effect of mixing and is independent of decay mode. Thus it may be referred to as  $CP$  violation purely in mixing.

In neutral  $B$  decays this effect is expected to be very small, since  $\Delta m = \mathcal{O}(10^3)\Delta\Gamma$ .  $CP$  violation purely in mixing should thus only enter the neutral  $B$  system at the  $10^{-3}$  level. An asymmetry in the measurements of the overall rate to flavor tagged  $B^0$  vs.  $\bar{B}^0$  would be a signature of  $CP$  violation purely in mixing. With larger statistical data samples, evidence for this may be seen; at present, experimental limits exist. It has been clearly observed, however, in the neutral kaon system (where it is the prevalent effect); the discovery of  $CP$  violation in 1964 was an observation of  $CP$  violation purely in mixing.

### 3.3 $CP$ Violation in Interference Between Decays of Mixed and Unmixed Mesons

Final states which may be reached from either  $B^0$  or  $\bar{B}^0$  decays can exhibit a third type of  $CP$  violation, which results from the interference between the decays of mixed and of unmixed neutral  $B$  mesons which both decay to the same final state (see fig. 2).

Consider the  $CP$ -violating asymmetry in rates between  $B^0$  and  $\bar{B}^0$  as a function of time:

$$a_{CP}(t) = \frac{\Gamma(B_{phys}^0(t) \rightarrow f) - \Gamma(\bar{B}_{phys}^0(t) \rightarrow f)}{\Gamma(B_{phys}^0(t) \rightarrow f) + \Gamma(\bar{B}_{phys}^0(t) \rightarrow f)} \quad (15)$$

Resolving the time-dependent rates  $\Gamma(t)$ , we obtain

$$a_{CP}(t) = C \cos(\Delta mt) - S \sin(\Delta mt) \quad (16)$$

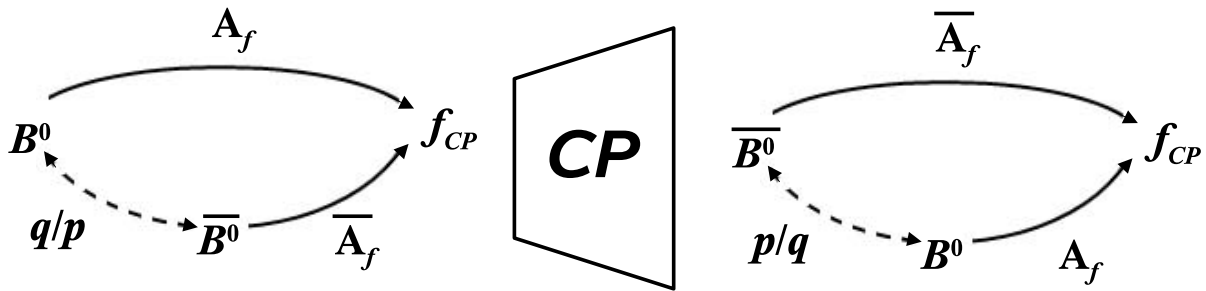


Figure 2: Effect of the “ $CP$  mirror” on  $B^0$  decay to a  $CP$  eigenstate  $f_{CP}$ . The  $CP$  asymmetry is due to the interference between mixing, described by parameters  $p$  and  $q$ , and the decay amplitudes  $A_f$  and  $\bar{A}_f$ .

where

$$C = \frac{1 - |\lambda|^2}{1 + |\lambda|^2} \quad (17)$$

$$S = \eta_{CP} \frac{-2 \sin(2(\phi_M + \phi_D))}{1 + |\lambda|^2} = \frac{2 \operatorname{Im} \lambda}{1 + |\lambda|^2} \quad (18)$$

and  $2\phi_M$  is the phase of  $q/p$ ,  $\phi_D$  is the phase of the decay,  $\eta_{CP}$  is the  $CP$  eigenvalue of  $f$ , and

$$\lambda = \frac{q \langle f | \mathcal{H} | \bar{B}^0 \rangle}{p \langle f | \mathcal{H} | B^0 \rangle} = \frac{q \bar{A}_f}{p A_f} = |\lambda| e^{-2i(\phi_M + \phi_D)}. \quad (19)$$

We assume in these expressions  $|q/p| \approx 1$ .

In the absence of  $CP$  violation,  $S$  and  $C$  must both go to zero, since they occur only when weak phases do not cancel.  $C$  is only nonzero when the ratio of the amplitude norms differs from unity, which is the signature of direct  $CP$  violation (detailed in section 3.1). However, it is possible that  $|q/p| = 1$  and  $|\lambda| = 1$ , *i. e.*, there is no  $CP$  violation in either mixing or decay, but the  $CP$  asymmetry in eq. 16 is nonzero, because  $\operatorname{Im} \lambda \neq 0$ . In this case from the definition in equation 18,  $S$  is non-zero. This represents a distinct type of  $CP$  violation. It results from the interference of the decays of mixed mesons with those of unmixed mesons (*CP violation in the interference between decay with and without mixing*, or *mixing-induced CP violation*). Unlike  $CP$  violation in decay, no nontrivial strong phases are required.

## 4 $CP$ Violation in the Standard Model

$CP$  violation within the context of the Standard Model  $SU(2) \times U(1)$  electroweak symmetry[22] was introduced by Kobayashi and Maskawa in 1973 via the postulation of a third family of quarks.[10] They suggested a generalization of the quark mixing matrix, introduced by Cabibbo[9], to describe transitions between quark generations through charged current ( $W^\pm$ ). This complex unitary matrix is called the Cabibbo-Kobayashi-Maskawa (CKM) matrix,

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \quad (20)$$

The unitarity of the matrix  $V$  and the freedom to arbitrarily choose the global phases of the quark fields reduce the initial nine unknown complex elements of  $V$  to three real numbers and one phase, where the latter accounts for  $CP$  violation. Because these four numbers effectively govern the rates of all tree- and loop-level electroweak transitions that involve the charged current, it is a compelling exercise to overconstrain  $V$ . If inconsistencies among different measurements occur, it would reveal the existence of physics beyond the SM.

### 4.1 Unitarity Conditions and the Unitarity Triangle

Unitarity of the CKM matrix  $V$  gives 9 independent equations. The equations for the off-diagonal elements, each containing a sum of 3 complex numbers which equals 0, will each describe a triangle



in the complex plane:

$$V_{cd}V_{ud}^* + V_{cs}V_{us}^* + V_{cb}V_{ub}^* = 0 \quad (21)$$

$$V_{cd}V_{td}^* + V_{cs}V_{ts}^* + V_{cb}V_{tb}^* = 0 \quad (22)$$

$$V_{ud}V_{td}^* + V_{us}V_{ts}^* + V_{ub}V_{tb}^* = 0 \quad (23)$$

$$V_{us}^*V_{ud} + V_{cs}^*V_{cd} + V_{ts}^*V_{td} = 0 \quad (24)$$

$$V_{ub}^*V_{us} + V_{cb}^*V_{cs} + V_{tb}^*V_{ts} = 0 \quad (25)$$

$$V_{ub}^*V_{ud} + V_{cb}^*V_{cd} + V_{tb}^*V_{td} = 0 \quad (26)$$

The differences between these 6 triangles are purely empirical. There is no theoretical motivation at present for the fact that 4 of them are nearly degenerate and only 2 describe triangles that have each of their sides being the same order of magnitude in length. It is an empirical fact that only eqs. 23 and 26 above describe triangles which are not nearly degenerate. The triangle corresponding to equation 26 is the one that is used to pictorially represent the irreducible  $CP$  violating phase and is referred to as the Unitarity Triangle (UT).

There are several parameterizations of the CKM matrix available in the literature. Following the observation of a hierarchy between the mixing angles, Wolfenstein[23] proposed an expansion of the CKM matrix in terms of the four parameters  $\lambda$ ,  $A$ ,  $\rho$  and  $\eta$  ( $\lambda \simeq |V_{us}| \approx 0.23$  being the expansion parameter, that is the Cabibbo parameter  $\lambda \equiv \sin \theta_C$ ), which is widely used in the literature:

$$V = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4) \quad (27)$$

with  $(\lambda, A, \rho, \eta)$  as the 4 real parameters describing the CKM matrix, the latter 3 being of order 1.

Unitary triangle obtained by eq. 26 can be rotated and scaled choosing a conventional phase in a way that  $V_{cb}^*V_{cd}$  is real, and so aligning related side to real axis, and dividing length of all sides for  $|V_{cd}V_{cb}^*|$  so length is normalized to 1. The triangle (show in fig. 3) will have two fixed vertices at  $(0,0)$  and at  $(1,0)$  and coordinates of the remaining vertices will depends by  $(\rho, \eta)$  corresponding to Wolfenstein's parameters. The length of the two sides and the three angles of out unitary triangle, denoted with  $\alpha, \beta$  and  $\gamma$ , are related to the  $CP$  violation measurements. These quantities are physical and can be measured from  $CP$  asymmetries in  $B$  decays. Consistency among different experimental values helps in the verification of the SM.

## 5 Experimental Overview

Exploring  $CP$  violation in the  $B$  system and its potential impact on the Standard Model, baryogenesis, and cosmology, requires copious production of  $B$  mesons, accurate measurement of the  $B$  time of flight and flavor, and reasonably low background in the reconstruction. There are several potential options for experiments which can fulfill these criteria, like hadron colliders ( $p\bar{p}$ ),  $e^+e^-$  colliders at the  $Z$ -pole, and symmetric and asymmetric  $e^+e^-$  colliders at  $\Upsilon(4S)$  energy. The current operating experimental facilities belong to the last group.

The  $\Upsilon(4S)$  resonance provides a very clean environment for  $B$  reconstruction, with a very favorable ratio of  $b\bar{b}$  production from  $e^+$  and  $e^-$  beams compared to lighter quark pairs ( $\sigma(b\bar{b})/\sigma(q\bar{q}) \simeq 0.28$ ).

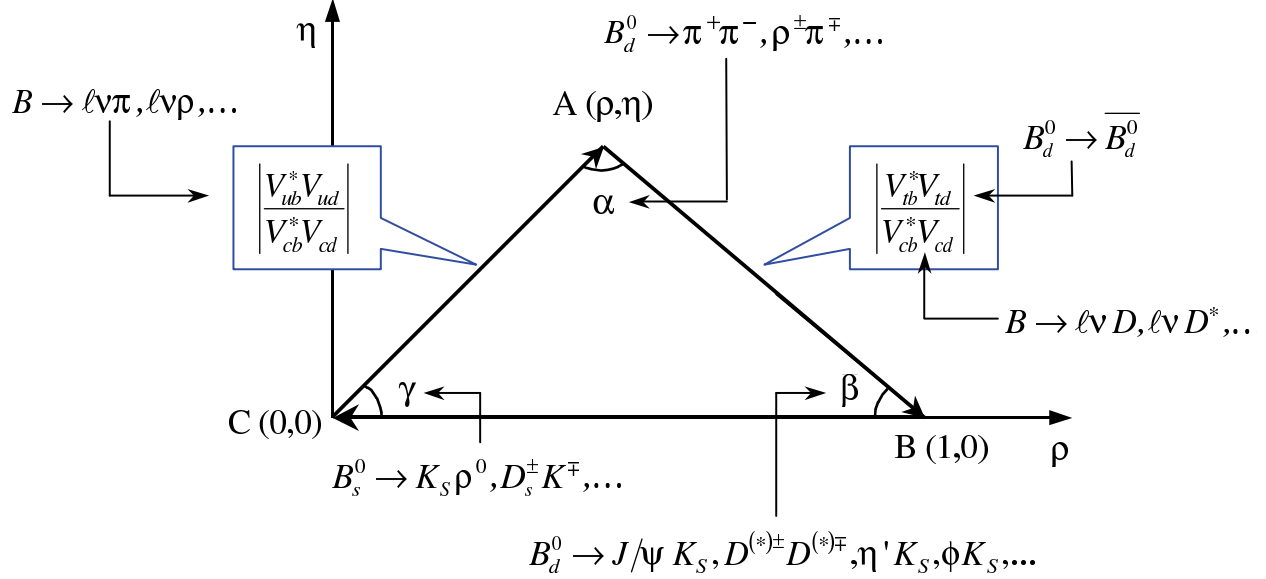


Figure 3: Unitary triangle and main decays to measure the sides and the angles.

Since for the  $CP$  asymmetry we need to measure the times of flight of the  $B$  mesons, we measure the positions of the  $B$  decay vertices in order to extract distances, and then times. In the case of symmetric  $e^+e^-$  beams the  $B$  mesons have a small boost ( $\gamma\beta = 0.06$ ) and they travel distances of the order of  $30 \mu m$ . These distances are too small to allow a time-dependent measurement of  $CP$  asymmetry. Asymmetric energy beams provide a boost to the  $B$  meson pair that is produced. In fact, unlike symmetric beams, the  $B$  particles are carried downstream in the direction of the higher energy beam and this forward boost enables the decay products to separate, allowing to observe the distances between their points of decay in lab frame.

Two asymmetric  $B$ -factories have been built and are currently producing physics: PEP-II[24] and KEK-B[25]. Previously, the symmetric collider CLEO (at the CESR ring at Cornell) was able to produce precision  $B$  physics results. However the symmetric design and the limited statistics precluded measurement of time-dependent  $CP$ -violating asymmetries.

The *BABAR* and Belle experiments are very similar, with the following main differences: the KEK-B/Belle  $B$  factory has a nonzero beam crossing angle ( $4.2 \text{ mr}$ ) at the interaction point (IP), whereas the PEP-II/*BABAR*  $B$  factory has a more traditional collinear IP. The KEK design allows a greater number of beam bunches to be stored in the ring, due to absence of parasitic crossings at  $\pm 1m$ , as are present in the PEP-II design. However KEK-B is a highly non-traditional design; concerns over higher-order mode resonances at the IP led the PEP-II  $B$  factory to use a collinear crossing. So far, both KEK-B and PEP-II have performed well. At the time of writing, PEP-II has integrated  $406.28 \text{ fb}^{-1}$  and KEK-B has integrated  $649.1 \text{ fb}^{-1}$ .

The particle identification method also differs between *BABAR* and Belle: *BABAR* uses quartz bars to internally reflect Cherenkov light to a backward-mounted detector (the DIRC), whereas Belle uses an aerogel Cherenkov detector. In addition, *BABAR* has a 5-layer silicon vertex detector that can do standalone tracking, whereas Belle uses a 3-layer silicon vertex detector. More details on *BABAR* and Belle detectors can be found in refs. [26] and [27], respectively.

## 5.1 Overview of Experimental Technique at the $\Upsilon(4S)$

$B^0$  and  $\bar{B}^0$  mesons produced by  $\Upsilon(4S)$  decay are in a coherent  $L = 1$  state ( $P$ -wave). One way to view this state is that each of the two particles evolve in time. However they evolve in phase, so that at any time, until one particle decays, there is always exactly one  $B^0$  and one  $\bar{B}^0$  present<sup>2</sup>. However once one of the particles decays the other continues to evolve, and thus there are possible events with two  $B^0$  or two  $\bar{B}^0$  decays, whose probability is governed by the time between the two decays. To measure  $CP$  asymmetries we look for events in which a  $B$  ( $B_{CP}$ ) decays in a  $CP$  eigenstate  $f_{CP}$  at  $t_{f_{CP}}$  time, while the other meson ( $B_{\text{tag}}$ ) decays in a way that allows us to identify its flavor, so called *tagging* mode, at  $t_{\text{tag}}$  time. For example, take a tagging mode where  $B_{\text{tag}}$  meson is identified as a  $B^0$  at  $t_{\text{tag}}$  time at which the tagging decay occurs. So at the same time  $B_{CP}$  is  $\bar{B}^0$ . After that  $B_{CP}$  evolves in time, decaying at time  $t_{f_{CP}} > t_{\text{tag}}$ . Note that this is true even when the tagging decay occurs after the  $CP$  eigenstate decay. In this case the state of the  $B_{\text{tag}}$  at any time  $t_{f_{CP}} < t_{\text{tag}}$  must be just that mixture which, if it had not decayed, would have evolved to become a  $B^0$  at time  $t_{f_{CP}} = t_{\text{tag}}$ . In this situation, we can evaluate time-dependent  $CP$  asymmetry that results to be equal to expression 16, where  $t = \Delta t = t_{f_{CP}} - t_{\text{tag}}$ :

$$a_{CP}(\Delta t) = C \cos(\Delta m \Delta t) - S \sin(\Delta m \Delta t). \quad (28)$$

This makes the measurements of time dependent  $CP$  asymmetry possible in the asymmetric  $B$ -factories because it is possible to obtain  $\Delta t$  from the reconstruction of the decay vertices of the two  $B$  mesons produced from  $\Upsilon(4S)$  decays.

## 5.2 Analysis Method

We reconstruct a  $B^0$  decaying into the  $CP$  eigenstates ( $B_{CP}$ ). From the remaining particles in the event we also reconstruct the decay vertex of the other  $B$  meson ( $B_{\text{tag}}$ ) and identify its flavor. The difference  $\Delta t \equiv t_{f_{CP}} - t_{\text{tag}}$  of the proper decay times  $t_{CP}$  and  $t_{\text{tag}}$  of the  $CP$  and tag  $B$  mesons, respectively, is obtained from the measured distance between the  $B_{CP}$  and  $B_{\text{tag}}$  decay vertices and from the boost ( $\beta\gamma = 0.56$  in  $BABAR$ ) of the  $e^+e^-$  system. The  $\Delta t$  distribution for the  $B \rightarrow f$  decay is given by:

$$F(\Delta t) = \frac{e^{-|\Delta t|/\tau}}{4\tau} [1 \mp \Delta w \pm (1 - 2w)(-\eta_{CP} S \sin(\Delta m_d \Delta t) - C \cos(\Delta m_d \Delta t))], \quad (29)$$

where  $\eta_{CP}$  is the  $CP$  eigenvalue of the final state  $f$ , the upper (lower) sign denotes a decay accompanied by a  $B^0$  ( $\bar{B}^0$ ) tag,  $\tau$  is the mean  $B^0$  lifetime,  $\Delta m_d$  is the  $B^0\bar{B}^0$  mixing frequency, and the mistag parameters  $w$  and  $\Delta w$  are the average and difference, respectively, of the probabilities that a true  $B^0$  is incorrectly tagged as a  $\bar{B}^0$  or vice versa.

We use the informations from the tracking system, the electromagnetic calorimeter, the Cherenkov detector, and the hadronic calorimeter to reconstruct charged and neutral particles in the final state. Two kinematic variables are used in general to discriminate between signal decays and combinatorial background. The first is  $\Delta E$ , the difference between the center-of-mass (CM) energy of the  $B$  candidate and the CM beam energy. The second is the beam-energy-substituted

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<sup>2</sup>This is yet one more particle physics case of the classic Einstein-Podolsky-Rosen situation.

mass  $m_{\text{ES}} \equiv \sqrt{(s/2 + \mathbf{p}_0 \cdot \mathbf{p}_B)^2/E_0^2 - \mathbf{p}_B^2}$ , where the  $B$  candidate momentum  $\mathbf{p}_B$  and the four-momentum of the initial  $\Upsilon(4S)$  state  $(E_0, \mathbf{p}_0)$  are defined in the laboratory frame.

Background events arise primarily from random combinations of particles in continuum  $e^+e^- \rightarrow q\bar{q}$  events ( $q = u, d, s, c$ ). We reduce these with requirements on shape-event variables, like the angle  $\theta_T$  between the thrust axis of the  $B$  candidate in the  $\Upsilon(4S)$  frame and that of the rest of the charged tracks and neutral calorimeter clusters in the event. Further discrimination against  $q\bar{q}$  background is also done with a Fisher discriminant  $\mathcal{F}$  or a neural network (NN) which combines several variables that characterize the production dynamics and energy flow in the event.[28] We study the background from other  $B$  decays using Monte Carlo (MC) simulated events.

The  $CP$ -violation parameters and signal yields are obtained from extended maximum likelihood fits with the input observables  $\Delta E$ ,  $m_{\text{ES}}$ ,  $\mathcal{F}$  or NN,  $\Delta t$  as well as the resonance mass and decay angle. In the fits, the likelihood for a given event is the sum of the signal, continuum and the  $B$ -background likelihoods, weighed by their respective event yields.

## 6 Results

In the following sections we will report three important results achieved by the  $B$ -factories. More informations can be found in ref. [4].

### 6.1 $CP$ Asymmetry Measurements in $b \rightarrow c\bar{c}s$ Amplitude

In  $b \rightarrow c\bar{c}s$  quark-level decays, the time-dependent  $CP$  violation parameters measured from the interference between decays with and without mixing are  $S$  and  $C$  defined in equation 18. In the SM, with a very good approximation, we expect for these decays  $S = -\eta_{CP} \sin 2\beta$  and  $C = 0$  for the transition  $B^0 \rightarrow f$ , where  $\eta_{CP} = \pm 1$  is the  $CP$  eigenvalue of  $f$  and  $2\beta$  is the phase difference between the  $B^0 \rightarrow f$  and  $B^0 \rightarrow \bar{B}^0 \rightarrow f$  decay paths. These modes are dominated by this single phase  $\beta = \arg(-V_{cd}V_{cb}^*/V_{td}V_{tb}^*)$  of the CKM mixing matrix, so  $\beta$  is the angle of the UT.

The theoretically cleanest case is  $B \rightarrow J/\psi K_{S,L}^0$ , but several other charmonium modes have been measured by BABAR and Belle:  $J/\psi K_S^0$ ,  $\psi(2S)K_S^0$ ,  $\chi_{c1}K_S^0$  and  $\eta_c K_S^0$  modes with  $\eta_{CP} = -1$ , as well as  $J/\psi K_L^0$ , which has  $\eta_{CP} = +1$ . In the latest result from Belle, only  $J/\psi K_S^0$  and  $J/\psi K_L^0$  are used. The world average reads[29]

$$\sin 2\beta = 0.674 \pm 0.026. \quad (30)$$

The first observation of  $CP$  violation outside the kaon system was announced by BABAR and Belle collaborations in 2002.[30] These measurements are consistent with the SM estimation, providing a crucial test of the mechanism of  $CP$  violation. As expected in the SM, no direct  $CP$  violation has been observed in all these modes.

### 6.2 $CP$ Asymmetry Measurements in $b \rightarrow s$ Amplitude

In the SM, decays of  $B^0$  mesons to charmless hadronic final states, such as  $\phi K^0$ ,  $f_0(980)K^0$ ,  $K^+K^-K^0$ ,  $\eta'K^0$ ,  $\pi^0 K_S^0$ ,  $K_S^0 K_S^0 K_S^0$ ,  $\rho^0 K_S^0$ ,  $\omega K_S^0$ , proceed mostly via a single loop (penguin) amplitude with the same weak phase as the  $b \rightarrow c\bar{c}s$  transition.[31] In these modes, assuming the penguin dominance of  $b \rightarrow s$  transition and neglecting CKM-suppressed amplitudes, the time-dependent  $CP$ -violation parameter  $S$  is expected to be  $\sin 2\beta$ , giving an independent measurement

of this parameter. However, CKM-suppressed amplitudes and the color-suppressed tree-level diagram, present in not pure-penguin modes (like  $f_0(980)K^0$ ,  $K^+K^-K^0$ ,  $\eta'K^0$ ,  $\pi^0K_S^0$ ,  $\rho^0K_S^0$ ,  $\omega K_S^0$ ), introduce additional weak phases whose contribution may not be negligible.[32, 33, 34, 35] As a consequence, only an effective  $S = \sin 2\beta_{\text{eff}}$  is determined. The deviation  $\Delta S = S - \sin 2\beta$  has been estimated in several theoretical approaches: QCD factorization (QCDF)[34, 36], QCDF with modeled rescattering[37], Soft Collinear Effective Theory (SCET)[38], and SU(3) symmetry[32, 33, 39]. The estimates are channel-dependent. QCDF and SCET models estimate  $\Delta S$  to be positive in the most of modes. SU(3) symmetry provides unsigned bounds, assuming the worst case for strong phase, of the order  $|\Delta S| \lesssim 0.05$  in the best case.

Due to the large virtual mass scales occurring in the penguin loops, the possible presence of additional diagrams with new heavy particles in the loop and new  $CP$ -violating phases may contribute to the decay amplitudes. In this case the measurements of significantly larger  $\Delta S$  are a sensitive probe for physics beyond the SM.[35] Therefore, the main interest in these modes is not simply to measure  $\sin 2\beta$ , but to search for the new physics.

In the fall of 2006, *BABAR* and Belle collaborations announced the observation of  $CP$  violation due to the interference between decays with and without mixing in one of the  $b \rightarrow s$  modes,  $B^0 \rightarrow \eta' K^0$ . [40] The world average reads[29]

$$\sin 2\beta_{\text{eff}} = 0.61 \pm 0.07. \quad (31)$$

This is an important achievement because it constraints the possible effect of new physics. The events have been reconstructed with  $K^0 \rightarrow K_S^0$  ( $\eta_{CP} = -1$ ) and  $K^0 \rightarrow K_L^0$  ( $\eta_{CP} = +1$ ). In fig. 4 we give the  $\Delta t$  and asymmetry projections for *BABAR* for a subset of the data for which the ratio between the likelihood of signal events and the sum of likelihoods of signal and background events (computed without the variable plotted) exceeds a mode-dependent threshold that optimizes the sensitivity.

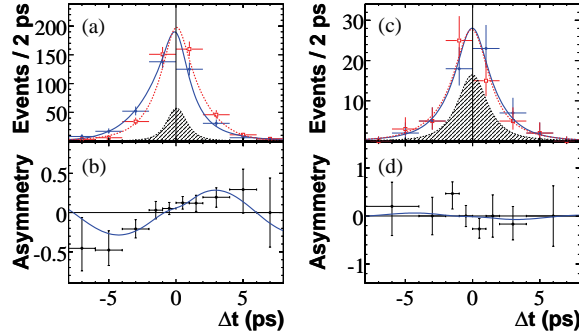


Figure 4: Projections (see text) onto  $\Delta t$  for *BABAR* for (a)  $\eta' K_S^0$  and (c)  $\eta' K_L^0$  of the data (points with error bars for  $B^0$  tags ( $N_{B^0}$ ) in red empty rectangles and  $\bar{B}^0$  tags ( $N_{\bar{B}^0}$ ) in blue solid circles), fit function (red dashed and blue solid lines for  $B^0$  and  $\bar{B}^0$  tagged events, respectively), and background function (black shaded regions). We show the raw asymmetry,  $(N_{B^0} - N_{\bar{B}^0}) / (N_{B^0} + N_{\bar{B}^0})$ , for (b)  $\eta' K_S^0$  and (d)  $\eta' K_L^0$ ; the lines represent the fit functions.

In general the individual  $\sin 2\beta_{\text{eff}}$  results are in agreement with the charmonium value for each mode. However, these measurements are statistically limited and more data are needed to give a clear conclusion. None of the modes studied exhibits non-zero direct  $CP$  violation.

### 6.3 Direct $CP$ asymmetry

The CKM mechanism causes “direct”  $CP$  violation in the decay, as soon as at least two amplitudes with different strong and weak phases contribute. Because virtual loops are present in all meson decays, “some” (possibly unobservable) amount of direct  $CP$  violation occurs. Owing to the large weak phases arising in  $B$  decays, direct  $CP$  violation should be more prominent here than, *e. g.*, in the kaon system. This has been confirmed by the measurement of the direct  $CP$  asymmetry  $A_{K^+\pi^-} = -0.093 \pm 0.015$  in  $B^0 \rightarrow K^+\pi^-$  decays[29]. Evidence for direct  $CP$  violation in neutral  $B$  decays also exists for  $B^0 \rightarrow \pi^+\pi^-$  ( $5.6\sigma$  significance)[29]. Also in this case it is possible to see how the  $B$ -factories have given an important contribution if you consider that the discovery of the direct  $CP$  violation in the kaon system occurs 35 years after the first observation of the  $CP$  violation in the mixing.

## 7 Conclusion

The measurements of  $CP$  violation in the kaon system have played an important role to understand the  $CP$  violation. However, the smallness of the  $CP$  violating effects in the kaon system is an impediment to progress in that sector. The present and the future to constraint the CKM scenario (or to find effects beyond the SM) is in general given by the decays with  $b$  quark. So far the  $B$ -factories have performed very well, giving an important contribution to our understanding of the  $CP$  violation. All main goals have been reached. Combining all results in a global CKM fit[41], we obtain that all results are consistent each other and consistent with the SM predictions. There is no clear evidence of contribution coming from physics beyond the SM. However, most of the current measurements are statistically limited and more data are needed to clarify the situation and new generations of experiments will provide the possibility to investigate better effects of new physics.

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